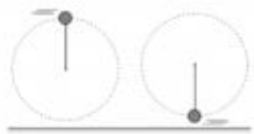
NAME ______________________ DATE ____________

Scenario

A ball whose weight is 2 N is attached to the end of a cord of length 2 m as shown. The ball is whirled in a vertical circle clockwise. The tension in the cord at the top of the circle is 7 N, and the tension at the bottom is 15 N. Two students discuss the net force on the ball at the top of the circle.



Dominique: "~~The net force on the ball at the top position is 7 N since the net force is the same as the tension.~~"

Carlos: "No, the net force on the ball includes ~~the centripetal force~~, tension, and weight. The tension and the weight are acting downward and have to be added. ~~Then you need to figure out the centripetal force~~ $\cancel{\left(\frac{mv^2}{r}\right)}$ ~~and include it in the net force.~~"

Analyze Data

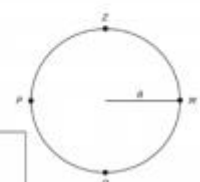
PART A: Cross out the incorrect statements for each student's argument.

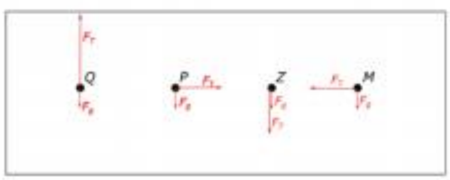
PART B: In a few short sentences, state the net force on the ball at the top of the circle and support your claim with evidence.

Carlos is correct that the net force includes the weight and the tension force and they both point down so that they will be added together. Therefore, the net force on the ball at the top of the circle is 7 N + 2 N = 9 N. These two forces together add to be the centripetal force.

Using Representations

PART C: The diagram at right shows the circular path of the ball from Part A. The dots below represent the ball at the marked locations on the circular path. Draw free-body diagrams showing and labeling the forces (not components) exerted on the ball at each point. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces.





Quantitative Analysis

PART D: Derive an expression for the minimum speed the ball can have at point Z without leaving the circular path. For each line in the derivation, explain what was done mathematically. The first line is completed for you as an example.

$\sum F = ma_c$	The sum of the force is equal to *ma*, and since the ball is in circular motion, *a* is the centripetal acceleration.
$\Sigma F_c = \frac{mv^2}{r}$	Centripetal acceleration can be represented by the velocity squared over radius.
$F_T - F_g = \frac{mv^2}{r}$	The two forces on the ball at the top of the circle (Point Z) are the force of tension and the gravitational force, both of which are exerted downward.
$mg = \frac{mv^2}{r}$	At the minimum speed, the tension goes to zero.
$v = \sqrt{gr}$	The masses cancel in the equation and the equation is rearranged to solve for velocity.

PART E: Suppose the ball breaks at point P. Describe the motion of the ball after the string breaks. (When describing the motion of an object, you need to discuss what is happening to the position, velocity, and the acceleration of the object.) Tell the story of the motion of the ball from the time the string breaks until the ball reaches the ground.

Position:

The ball travels straight up to a maximum height and then comes straight back down.

Velocity:

The speed of the ball decreases until it reaches the maximum height at which point the ball turns around and speeds back up until it hits the ground.

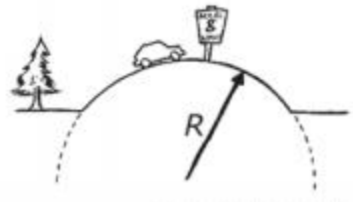
Acceleration:

The acceleration of the ball is always 9.8 m/s² down after the string is cut.

NAME ____________________ DATE ____________

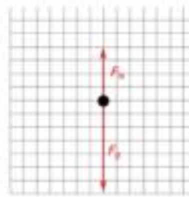
Scenario

A car of mass m passes over a bump in a roadway that follows the arc of a circle of radius R as shown.



Using Representations

PART A: The dot, at right below the picture, represents the car at the top of the hill. Draw a free-body diagram showing and labeling the forces (not components) exerted on the car. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces. Each force must be represented by a distinct arrow starting on and pointing away from the dot.



Quantitative Analysis

PART B: Starting with Newton's second law, derive an expression for the maximum speed v the car can have without losing contact with the road. For each line of the derivation, explain what was done mathematically (i.e., annotate your derivations). Your expression should be in terms of R and physical constants.

$\sum F = ma$	The net force on the car at the top of the hill is equal to the acceleration of the car times the mass of the car.
$F_g - F_N = \frac{mv^2}{r}$	There are two forces exerted on the car at the top of the hill: the gravitational force downward and the normal force upward.
$F_g - 0 = \frac{mv^2}{r}$	If the car is going the maximum speed, the normal force is equal to zero.
$F_g = \frac{mv^2}{r}$	Therefore, the gravitational force is the only force contributing to the centripetal force.
$mg = \frac{mv^2}{r}$	The masses cancel.
$v_{max} = \sqrt{gr}$	And the maximum speed is equal to:

Argumentation

PART C: In your derivation, you set the normal force equal to zero. Explain why.

The maximum speed for the car will be when the centripetal force is equal to the weight of the car. At that point, the weight is the only force exerted on the car. If the car goes too fast, it will lose contact with the road, so as it approaches the maximum speed, the normal force approaches zero.

PART D: A truck of mass 2 m passes over the same bump. Compared to the car, how many times bigger or smaller is its maximum speed without losing contact with the road? Justify your answer with reference to your expression in Part B.

The maximum speed is the same regardless of the mass of the vehicle. The mass cancels out of the equation.

Checklist:

- ______ I answered the question directly.
- ______ I stated a law of physics that is always true.
- ______ I connected the law or laws of physics to the specific circumstances of the situation.
- ______ I used physics vocabulary (force, mass, acceleration, centripetal, velocity, speed, time, radius).

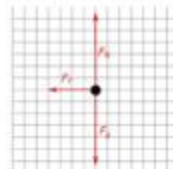
NAME ____________________ DATE ____________

Scenario

A police car of mass m moves with constant speed around a curve of radius R. (The car is, from your point of view, coming out of the page and is in the process of turning towards the left side of the page.) The car is moving as fast as it can without sliding out of control on the flat roadway to respond to an emergency. This maximum safe speed is v_0. The coefficient of static friction between the car's tires and the roadway is μ_0.

Using Representations

PART A: The dot at right represents the car. Draw a free body diagram showing and labeling the forces (not components) exerted on the car as it rounds the corner. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces. Each force must be represented by a distinct arrow starting on and pointing away from the dot.



Argumentation

PART B: i. Suppose that the car encounters a wet section of the curved roadway so that this section of the curve has a coefficient of friction less than μ_0. The maximum safe speed to make this turn is v_1. Mark the correct relationship between v_0 and v_1.

__X__ $v_1 < v_0$ ______ $v_1 = v_0$ ______ $v_1 > v_0$

Explain your reasoning using physical principles without manipulating equations. (This means you may reference equations from the equation sheet but should not derive an equation for the relationship between μ and F_N.)

The friction force provides the centripetal force. If there is a smaller coefficient of friction and the mass remains constant, there will be a smaller magnitude friction force changing the direction of motion, so the direction must change slower. If the radius stays the same, v_1 must be smaller than v_0

ii. Suppose that the police car arrives at another section of roadway that also curves but has a radius of curvature greater than R. The maximum safe speed to make this turn is v_2. Mark the correct relationship between v_0 and v_2.

______ $v_2 < v_0$ ______ $v_2 = v_0$ __X__ $v_2 > v_0$

Explain your reasoning using physical principles without manipulating equations. (This means you may reference equations from the equation sheet but should not derive an equation for the relationship.)

With a larger radius, the car must go farther to change its direction the same amount. Therefore, the car can go faster with the same force changing its direction.

Quantitative Analysis

PART C: Derive an expression for the maximum safe speed that the car can take the turn in terms of μ and R.

$\Sigma F_c = ma_c$	The sum of the forces in the horizontal direction is equal to the mass of the car times the acceleration of the car. In this case, the acceleration is centripetal acceleration
$\Sigma F = \frac{mv^2}{r}$	The centripetal acceleration of the car is equal to speed squared over the radius of the turn
$F_f = \frac{mv^2}{r}$	In this case, it is the friction force that provides the centripetal force
$\mu mg = \frac{mv^2}{r}$	The friction force is equal to the coefficient of friction times the normal force
$\mu g = \frac{v^2}{r}$	The masses cancel
$v = \sqrt{\mu g R}$	The maximum speed of the car around the circle of radius R and coefficient μ is:

PART D: i. Explain how your expression in Part C supports your answer for Part B (i).

A smaller coefficient of friction under the radical makes the right side of the equation smaller, so v must be smaller than before.

ii. Explain how your expression in Part C supports your answer for Part B (ii).

R is in the numerator, so a larger radius means a higher limit on v.

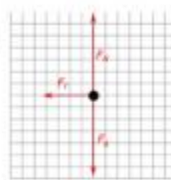
NAME ______________________ DATE ____________

Scenario

Consider a coin of mass m placed on a rotating surface a distance R from the axis of rotation. The surface rotates with a period T. There are some locations on the surface where the coin can be placed and the force of static friction will not allow the coin to slip. At other locations, the coin will slip because static friction is not strong enough to prevent the coin from slipping. The coefficient of static friction between the coin and the surface is μ.

Using Representations

PART A: The dot at right represents the coin when the coin is at the location shown above in the diagram. Draw a free-body diagram showing and labeling the forces (not components) exerted on the coin. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces. Each force must be represented by a distinct arrow starting on and pointing away from the dot.



Create an Equation

PART B: Starting from the equation $F_f \le \mu F_N$, an inequality has been derived that must be satisfied at all times that the coin does not slip on the surface. The derivation has been done for you. You must fill in the annotations to explain each step.

$F_f \le \mu F_N$	The static friction force by definition is less than or equal to the coefficient of static friction times the normal force.
$F_f \le \mu mg$	In this case, the normal force is equal to the weight of the coin.
$\frac{mv^2}{R} \le \mu mg$	And the friction force is providing the centripetal force, so we can set F_f equal to the centripetal force.
$\frac{v^2}{R} \le \mu g$	Mass cancels.
$v^2 \le \mu gR$	The speed of the coin on the rotating surface will depend on the radius, but at all positions on the table, the time to make one complete circle remains constant.
$v \le \sqrt{\mu gR}$	Substituting $\frac{2\pi R}{T}$ for the speed of the coin
$\frac{4\pi^2 R}{T^2} \le \mu g$	The relationship between the coefficient of friction and the radius can be expressed as:

Argumentation

Blake and Carlos are trying to predict whether the coin will slip if the coin is "too close" to or "too far" from the axis of rotation. The students reason as follows:

Blake: "I think that the coin will slip if it is too close to the axis. It is like if a car takes a turn too tightly, the car can slide out of control. There's not enough force if the radius is too small.

Carlos: "I think that the coin will slip if it is too far from the axis. It's like a merry-go-round; if I ride a merry-go-round near the center, then I don't feel much force pulling me to the outside, but if I ride near the outside, there is more force pulling me away from the axis.

PART C: For each student's statement, state whether the inequality written in Part B provides support for that statement. If so, explain how. If not, explain why not. Ignore whether the student's statement is correct or incorrect for this part.

Blake's Statement	***Carlos's Statement***
If R is small, then the left side of the equation will be small, meaning that the coin is not likely to slip, as it will be less than or equal to the coefficient of friction times the acceleration due to gravity.	However, if R is large, then the left side of the equation will be large (since R is in the numerator), making it likely that there will not be enough friction force to keep the coin in that circle, and it could slide off.

PART D: State whether the coin will slip when it is "too close" to or "too far" from the axis.

_____ too close __X__ too far

PART E: Angela and Dominique are arguing over how the mass of the coin affects whether it will slip or not. Angela believes that a lighter coin is less likely to slip because a lighter coin requires less force. Dominique believes that a heavier coin is less likely to slip because a heavier coin can have a greater amount of friction. Using your equations along with other physical principles, explain how the coin's mass affects its likelihood of slipping.

Since the mass canceled out during the derivation and the final expression does not include the mass, the mass of the coin does not affect whether it will slip. Since the force that causes the motion is proportional to the mass then the force increases in the same way as the resistance to the force, so the motion will be the same, independent of the mass.

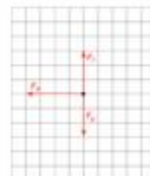
NAME ____________________ DATE ____________

Scenario

Carlos (mass m) enters the carnival ride called the "Rotor." The ride begins to rotate, and once Carlos has reached speed v, the floor drops out and he does not slip.

Using Representations

PART A: The dot at right represents the student on the ride after the floor has dropped out. Draw a free-body diagram showing and labeling the forces (not components) exerted on the student. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces. Each force must be represented by a distinct arrow starting on and pointing away from the dot.



Create an Equation

PART B: Derive an equation for the normal force on Carlos after the floor has dropped out. For each line of the derivation, explain what was done mathematically (i.e., annotate your derivation). Express your answer in terms of m, v, R and physical constants as appropriate.

$\Sigma F_c = ma_c$	The sum of the forces in the horizontal direction is equal to Carlos's mass times his centripetal acceleration
$F_N = \frac{mv^2}{R}$	The only force on Carlos horizontally is the normal force from the wall pushing him in toward the center of the circle.

Data Analysis

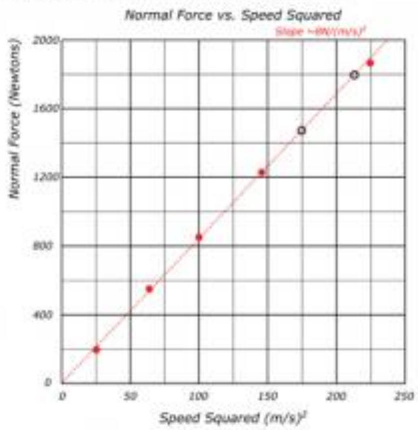
On the next ride, Carlos takes a force sensor and places it between himself and the wall of the ride and collects the following data about the force from the wall and the speed of the ride:

Force from the Wall (N)	*Speed of the Ride* (m/s)	Speed of the Ride Squared $(m/s)^2$
190	5	25
540	8	64
840	10	100
1,225	12	144
1,850	15	225

PART C: Which quantities should be graphed to yield a straight line whose slope could be used to determine the radius of the ride? Justify your answer. You may use the remaining columns in the table above, as needed, to record any quantities (including units) that are not already in the table.

Carlos should graph the normal force from the wall vs. the speed squared.

PART D: Plot the graph on the axes below. Label the axis with the variables used and appropriate numbers to indicate the scale. Draw a best-fit line and find the slope of the line.



$$slope = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1{,}800\text{ N} - 1{,}480\text{ N}}{212.5\left(\frac{m}{s}\right)^2 - 175\left(\frac{m}{s}\right)^2} = \frac{320\text{ N}}{37.5\left(\frac{m}{s}\right)^2} = 8.5\frac{\text{N}}{\left(\frac{m}{s}\right)^2}$$

PART E: Using the slope calculated in Part D, determine the radius of the ride if Carlos's mass is 50 kg.

The slope of the line is approximately 8.5 N/(m/s)². This will be equal to Carlos's mass divided by the radius of the ride. So, 8.5 N/(m/s)² = 50 kg/R. R should be approximately 6 meters.

NAME ______________________ DATE ____________

Scenario

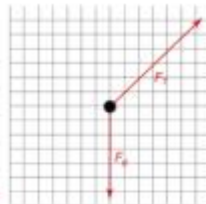
Consider a ball of mass M connected to a string of length L. A student holding the free end of the string whirls the ball in a horizontal circle with constant speed. The angle between the string and the vertical is θ. The student attempts to whirl the ball faster and faster in order to make the string become horizontal. No matter how fast the student whirls the ball, the string is never exactly horizontal.

Using Representations

PART A: i. The dot below represents the ball at the instant it appears in the diagram. Draw a free-body diagram showing and labeling the forces (not components) exerted on the ball. Draw the relative lengths of all vectors to reflect the relative magnitudes of all the forces.

ii. By discussing specific features of your force diagram, explain why the rope cannot become completely horizontal no matter how fast the ball is whirled.

Because the gravitational force is always pointing down, there must always be some vertical component of the tension to keep the ball in vertical equilibrium. Whirling the ball faster increases the horizontal component of the tension, but with any vertical component, F_T is never horizontal.



Create an Equation

PART B: i. Derive an equation that relates the speed v of the ball in its circle to the string length L and angle θ. [Hint: What force component provides the centripetal acceleration? How can you find the radius of the circle in terms of L and θ?]

$\Sigma F_y = ma_y$	The sum of the forces in the vertical direction is equal to ma_y
$F_T\cos\theta - mg = ma_y$	In the vertical direction, there is one component of the force of tension upward and the gravitational force downward. (We know to take the components of the Tension force here because the acceleration of the ball is directly in toward the center; therefore, parallel and perpendicular to that direction are our axes.

$F_T \cos\theta - mg = 0$	Since the ball is not accelerating in the vertical direction, this acceleration is zero.
$F_T \cos\theta = mg$	So, the component of the tension in the vertical direction is equal in magnitude to the gravitational force.
$F_T = \frac{mg}{\cos\theta}$	We can write an expression for the force of tension in terms of the angle and the weight of the ball:
$\Sigma F_x = ma_x$	In the horizontal direction, the sum of the forces is equal to ma_x, and this acceleration is the centripetal acceleration.
$F_T \sin\theta = \frac{mv^2}{r}$	And it is the horizontal component of the force of tension that provides the centripetal force.
$\frac{mg}{\cos\theta}\sin\theta = \frac{mv^2}{r}$	Plugging in what we derived earlier for the force of tension:
$g\tan\theta = \frac{v^2}{r}$	Sine θ/cosine θ is equal to tangent θ, and the masses cancel.
$g\tan\theta = \frac{v^2}{L\sin\theta}$	The radius of the circle is not the length of the string but is equal to $L\sin\theta$.
$v = \sqrt{gL\sin\theta\tan\theta}$	Solving for v:

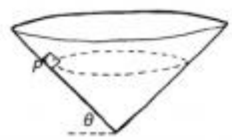
ii. How does your equation in Part B (i) show that the rope cannot become horizontal no matter how fast the ball is whirled?

If θ is 90°, the tangent of 90 is undefined (meaning in physics it goes to infinity), so v would have to be infinitely large, which is impossible.

NAME ____________________ DATE ____________

Scenario

Consider a cone made of a material for which friction may be neglected. The sides of the cone make an angle θ with the horizontal plane. A small block is placed at point P. In Case 1, the block is released from rest and slides down the side of the cone toward the point at the bottom. In Case 2, the block is released with initial motion so that the block travels with constant speed along the dotted circular path.



Data Analysis

PART A: In Case 1, the block is released from rest. Is the block accelerating?

X Yes ____ No

Explain, and if yes, determine the direction of the acceleration.

The block slides down the side of the cone from point P toward the point of the cone. Since there is no friction, the block accelerates down the cone, parallel to its surface.

In Case 2, the block is released so that it travels with a constant speed along the dotted circular path. Is the block accelerating?

X Yes ____ No

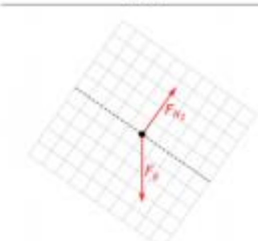
Explain, and if yes, determine the direction of the acceleration.

Yes, the block is still accelerating! This time, although the block is moving at a constant speed, it is traveling in the circle, so it has a centripetal acceleration toward the center of the circle.

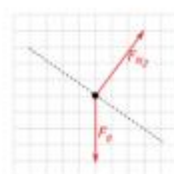
Using Representations

PART B: In both diagrams below, the weight F_g of the block is drawn. Draw the normal force exerted in each case on the corresponding diagram. Use the grids provided to make each normal force have the proper length. (In each case, breaking one of the forces into components will help you find the direction of the acceleration.)

Case 1



Case 2



Quantitative Analysis

PART C: Derive an expression for the magnitude of the normal force exerted on the object in each case in terms of F_g, θ, and physical constants as necessary.

Case 1	*Case 2*
"Where the 'y' direction is defined as perpendicular to the surface of the cone." $\Sigma F_y = ma_y$ $F_N - F_g \cos\theta = ma_y$ $F_N - F_g \cos\theta = 0$ $F_N = F_g \cos\theta$	"Where the 'y' direction is defined to be straight up and down." $\Sigma F_y = ma_y$ $F_N \cos\theta - F_g = ma_y$ $F_N \cos\theta - F_g = 0$ $F_N \cos\theta = F_g$ $F_N = \frac{F_g}{\cos\theta}$

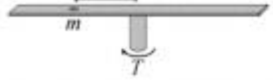
PART D: Use the diagrams in Part B to explain why the normal force is greater in Case 2. Then use your equations in Part C to explain why the normal force is greater in Case 2.

In Case 1, the normal force only cancels the component of the weight perpendicular to the plane—the normal force is equal to only a part of the weight force. (The other part of the weight contributes to the acceleration of the block down the incline.) So, in Case 1, the normal force is less than the weight. In Case 2, only one component the normal force cancels the weight. (The other component of the normal force contributes to the centripetal acceleration of the block.) So, in this case, the normal force is greater than the weight. In Case 1, the normal force is equal to $F_N = W\cos\theta$, weight times a fraction, and in Case 2, the normal force is equal to $F_N = \frac{W}{\cos\theta}$, or weight divided by a fraction. The normal force is greater in Case 2.

NAME ______________________ DATE ____________

Scenario

A student is attempting to determine the coefficient of static friction μ_s between a coin and a steel plate. The student attaches the center of the plate to a freely rotating axis. For each trial, the student sets the coin on different positions on the steel plate and measures the distance r from the center of the coin to the center of the axis of rotation. The student also has a stopwatch to measure the period T of the plate's rotation.



Experimental Design

PART A: Explain how the student can use this setup to take measurements that would allow the coefficient of static friction to be calculated. Be sure to explain clearly what rotational period must be measured and how experimental error can be reduced.

The student should gradually speed up the rotation of the plate until the coin slips. Then time 10 full rotations and divide by 10 to get the period. Repeat this several times.

Quantitative Analysis

PART B: Starting with Newton's laws and basic equations for circular motion, derive an equation that relates μ, r, T, and fundamental constants.

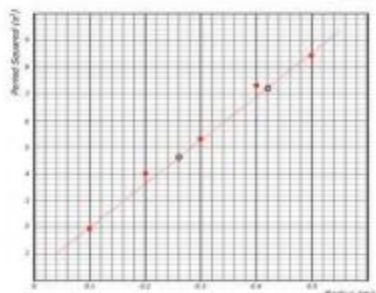
$\Sigma F_c = ma_c$	The sum of the forces on the coin in the horizontal direction are equal to the mass of the coin times the coin's acceleration
$F_f = \frac{mv^2}{r}$	In this case, it is the force of friction that provides the centripetal force for the coin
$\mu mg = \frac{mv^2}{r}$	And in this case, the normal force is equal to the gravitational force, so
$\mu gr = v^2$	the speed squared is equal to:
$\mu gr = \frac{4\pi^2 r^2}{T^2}$	The speed will change depending on where on the plate the coin is placed, but the period (T) that it takes for the coin to go around once remains the same regardless of where the coin is placed. The speed and the period are related by $v = \frac{2\pi r}{T}$

$\mu g = \frac{4\pi^2 r}{T^2}$	Dividing both sides by r.
$T^2 = \frac{4\pi^2 r}{\mu g}$	T squared is proportional to r.

The student collects the data shown in the table above.

PART C: Plot the data on the T^2 vs. r graph shown below. Draw a best-fit line to the data and calculate the slope of the best-fit line.

r (m)	T (s)	T^2 (s^2)
0.1	1.4	1.96
0.2	2.0	4.00
0.3	2.3	5.29
0.4	2.7	7.29
0.5	2.9	8.41



PART D: Use your equation from Part B along with the slope of your best-fit line from Part C to calculate the value of μ.

$$slope = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7.2\ s^2 - 4.6\ s^2}{0.42\ m - 0.26\ m} = \frac{2.6\ s^2}{0.16\ m} = 16.3\frac{s^2}{m}$$

$$slope = \frac{4\pi^2}{\mu g} = 16.3\frac{s^2}{m}$$

$$\mu = \frac{4\pi^2}{\left(16.3\frac{s^2}{m}\right)g} = 0.28$$